ANALYSIS OF A CIRCULAR WAVEGUIDE CAVITY LOADED WITH THICK FERRITE DISKS

J.M. Hellums
L.E. Davis

Department of Electrical Engineering, Rice University, Houston, Texas 77001

Abstract

Solutions with $e^{\pm j\theta}$ azimuthal dependence are investigated, with the aid of the computer, for a circular waveguide cavity containing thick, axially magnetized ferrite disks. Such an investigation and investigations of related configurations may have an application in the theory of the E-plane circulator.

In the analyses of waveguide circulators, more attention has been given to the H-plane device than to its E-plane counterpart. This is largely due to the separation of modes which is possible in the H-plane configuration, but impossible in the E-plane device. The usual geometry of an E-plane circulator[1,3] is shown in Fig. 1, and, in view of the complication of the boundary value problem, some simplification is desirable. The related configuration considered is illustrated in Fig. 2, in which the complicated boundary conditions $r = r'$ in Fig. 1 are replaced by a perfectly conducting wall, while the transverse ferrite-dielectric interfaces are retained. This is a circular waveguide cavity end-loaded with thick, axially magnetized ferrite disks, and a treatment of this problem, except with disks that are very thin or very nearly isotropic, has to our knowledge not appeared in the literature.

Determination of the fields in such a structure begins with investigation of circular guide filled with ferrite or dielectric. In each case we obtain an infinite number of possible modes which can be superimposed to give the actual fields in each region. For guide filled with ferrite, determination and description of these modes is difficult. Given the various parameters of the problem, the characteristic equation for the propagation constants must be solved numerically, and classification of the resulting modes is subtle.

Only the case of $e^{\pm j\theta}$ azimuthal dependence is considered at this stage. After taking into account the boundary conditions at the ends of the cavity, the fields in each region are expanded in the appropriate modes. Requiring continuity of the transverse field components at the ferrite-dielectric interfaces and making use of orthogonality of the dielectric modes then lead to a set of homogeneous simultaneous equations in the various arbitrary constants involved. In order to obtain a non-trivial solution, the determinant of these equations must be set to zero. This condition determines the cavity resonant frequencies. However, the number of simultaneous equations is in general infinite and it is convenient to assume that only a small number of modes is needed in each region to approximate the actual fields satisfactorily. The problem is further made more tractable by taking advantage of longitudinal symmetry.

The expressions for the modes in ferrite-filled guide with axial magnetization are complicated, and the characteristic equation for β , which is involved in each expression, must be solved numerically. This numerical solution is carried out with the aid of a graphical analysis that provides approximate information about the ferrite modes and makes the exact treatment feasible. For the $e^{\pm j\theta}$ cases, Suhl and Walker[2] (SW) have carried out such a numerical-graphical analysis. Each of their mode diagrams corresponds to a particular set of normalized constants associated with fixed values of saturation magnetization, gyromagnetic ratio, guide radius, ferrite permittivity and frequency. Such a diagram reveals the manner in which the propagation constants of the modes are influenced by the magnetic bias field. An extended version of the SW analysis of the ferrite-filled guide has been fundamental in this work. The extension is necessary not only because no SW plot applies to our particular combination of parameters, but also because the SW analysis only concerns the modes in their propagating forms, no information being given by the plots about a mode when it is evanescent. To consider modes in the latter situation, the characteristic equation must be examined for propagation constants that are imaginary. This leads to an expansion of the SW allowed regions and extensions of the solution curves. Further, situations arise in which quantities involved in the characteristic equation become complex, and investigation of that equation is further complicated.

For each of several frequencies over the range of interest, we must produce such an expanded mode plot. It is felt that some of the evanescent modes may be of substantial importance in the cavity problem. Inclusion of only the propagating modes in the modal expansions for each region may not permit good matching of fields at the ferrite-dielectric interfaces. Physically, we expect generation of evanescent modes at these interfaces, and we feel the theory must be capable of taking the most important of these into account.

Two representative mode plots are shown in Figs. 3 and 4, for frequencies of 8 GHz and 11 GHz respectively, and for the following constant parameters: azimuthal dependence $e^{j\theta}$, saturation magnetization ($4\pi M$) of 1800 Gauss; effective g-factor of 2.55; relative permittivity of 9.5; and guide radius of .587 cm. The normalized constants are defined as $p = 1.4 \cdot g_{eff} \cdot (4\pi M)/f$ and $r_0 = r'\beta_0$, where f is the frequency, r' is the actual guide radius, and β_0 is the free space propagation constant. The variable σ is a normalized form of the DC magnetic field and is given by $1.4 \cdot g_{eff} H_{DC}/f$. The variable λ_1 is defined in SW, is a function of σ, p and the propagation constant β, and for brevity is not given or discussed here. The characteristic equation for the filled guide can be expressed in terms of two functions, G_1 and G_2, which are equal when the equation is satisfied. For fixed r_0 and p , both G_1 and G_2 can be written as functions of λ_1 and σ. For various σ we are interested in determining the values of λ_1 that lead to equality of G_1 and G_2. Once a solution λ_1 is known for a given σ, β can be calculated.

As β varies from $+\infty$ to zero and then from zero to $+j\infty$, the value of λ_1 varies over a range which depends on σ. Taking the dependence on σ of this range into account an allowed region in the λ_1, σ plane is determined. This region is made up of the portions of the plane above and below both of the curves O_c and A. It should be mentioned that not all σ,β pairs map into the allowed region, since below the intersection of O_c and $(I_A')_T$ some imaginary values of β lead to complex values of λ_1. At present, however, there has been no indication of the existence of solutions corresponding to such values of β.

We now imagine the functions G_1 and G_2 to be represented by two surfaces that extend above and below the allowed portions of the λ_1,σ plane. Projections onto the λ_1,σ plane of intersections of these surfaces are solution curves. In most cases, the following procedure can be used to locate these curves approximately. Infinity and zero curves for both G_1 and G_2 are plotted in the allowed region. If it is known in which regions G_1 and G_2 are of like sign, continuity considerations can be used to verify the existence there of a solution curve segment. Consider, for example, the region enclosed by I_1, O_1 and $(I_B^{\prime})_T$. We can show that G_1 and G_2 are both positive in this region. For any positive value of G_1, a constant G_1 curve extends between I_1 and O_1 from the point $\lambda_1 = 1$, $\sigma = 1$ to $(I_B^{\prime})_T$. As one moves along this curve, G_2 moves through all positive values, including the chosen value of G_1. A solution point is therefore known to exist. This can be done for all the positive values of G_1, and a solution curve results. This solution curve must pass through the intersections $(O_\infty^{\prime})_T$, O_1 and $I_1,(I_B^{\prime})_T$.

In figures 3 and 4, much detail has been omitted in the crowded regions and only a few solution curves are shown. As an example of the conclusions that can easily be drawn from such plots, we note some effects of frequency on the modes. Only points above or below both O_c and $(I_A^{\prime})_T$ correspond to real values of the propagation constant. We then see in Fig. 3 that only one mode propagates for values of σ greater than 1.75 while for 11 GHz there are two modes for all σ greater than one. Note that for the circular guide filled with a dielectric of the permittivity mentioned above, only the TE_{11}^{o} mode propagates at 8GHz, while both the TE_{11}^{o} and TM_{11}^{o} modes propagate at 11 GHz. This is consistent with our conclusion from each plot about the number of modes propagating as σ approaches infinity. It is interesting to note theoretically, however, that as σ approaches zero in Fig. 4, only one mode propagates. This is not unreasonable, since the graphical treatment assumes that the magnetization of the ferrite remains constant as the bias magnetic field approaches zero. That is, the material is assumed to remain saturated.

The various computer methods for solution of the cavity characteristic equation and determination of the cavity fields require quantitative knowledge of the ferrite modes at many frequencies, and this knowledge cannot be obtained without the aid of the general information provided by the mode graphs. Using general expressions for the mode components in the ferrite and dielectric regions, the procedure for obtaining the simultaneous equations and the matrix whose determinant must be set to zero has been carried out.

Several possible extensions of this work are being considered and more results will be available in the next few months. The analysis can be modified to include the case of circular guide with a magnetically short circuiting wall. This modification can then be applied to disk-loaded cavities with curved magnetic walls instead of curved electric walls, and this may also prove useful in E-plane circulator theory. An extension of the ferrite-filled circular guide theory for both electrically and magnetically short circuiting walls to modes with higher order ($|n| > 1$) azimuthal dependence can be made, and applied to cavity problems and again, possibly, to the E-plane circulator. Also, cavities of both classes with ferrite disks in other positions can be treated.

References

1. Wright, W.H. and McGowan, J.W.: "High-Power Y-Junction E-Plane Circulator," IEEE Trans. Microwave Theory Tech., MTT-15, pp. 557-558, August 1968.

2. Suhl, H. and Walker, L.R.: "Topics in Guided Wave Propagation Through Gyromagnetic Media," Pt. 1, Bell System Technical Journal, vol. 33, pp. 579-659, May 1954.

3. Longley, S.R.: "Experimental 4-Port E-Plane Junction Circulators," IEEE Trans. Microwave Theory Tech. (Correspondence), MTT-15, pp. 378-380, June 1967.

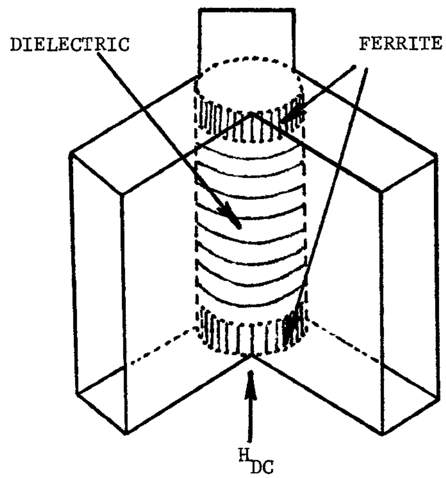

FIG. 1 The E-Plane Waveguide Circulator Structure

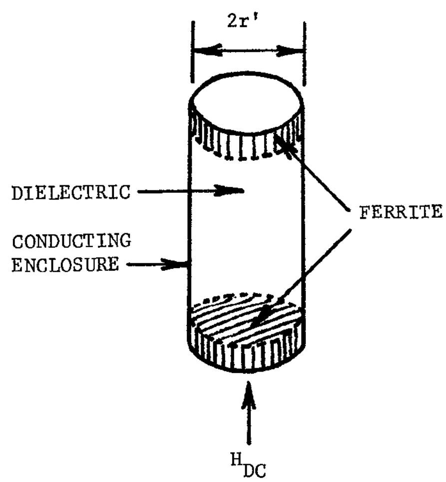

FIG. 2 The Cavity Structure for the "Reduced" Boundary Value Problem

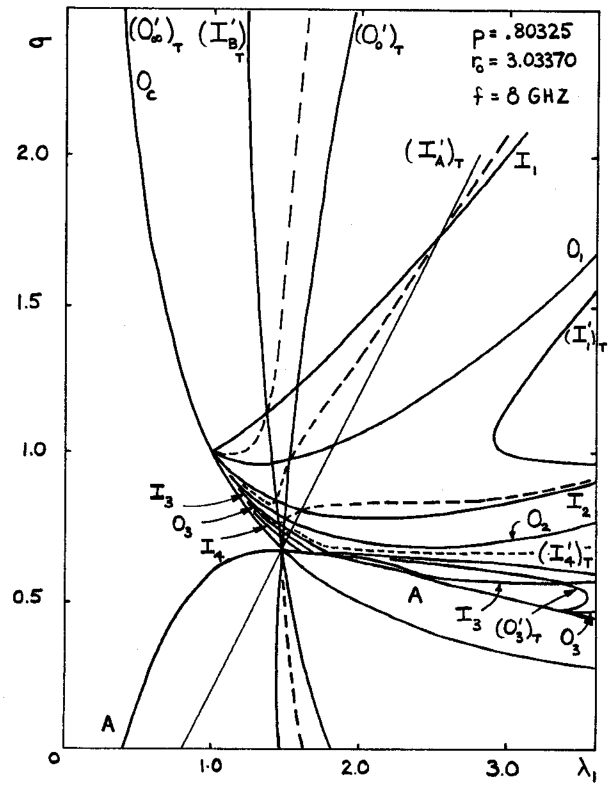

FIG. 3 Mode Chart for the Ferrite-Filled Waveguide at 8 GHz (other constants given in text)

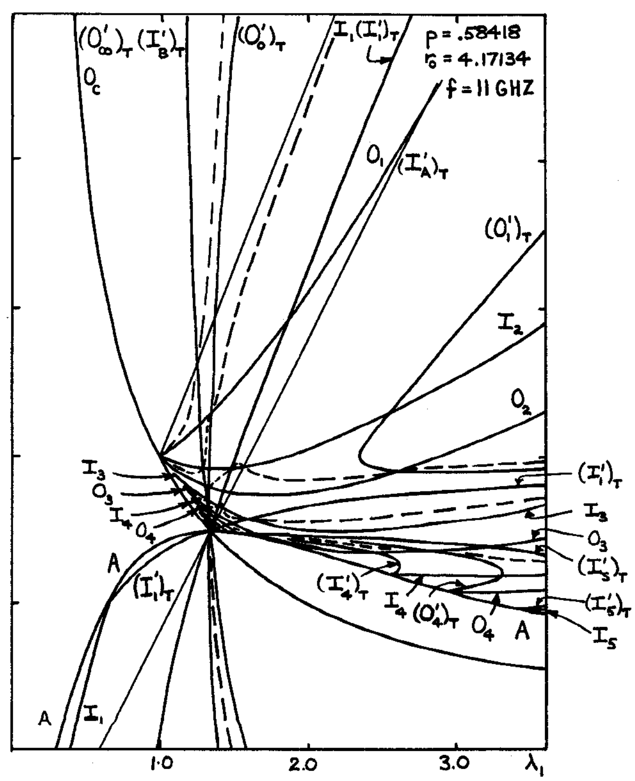

FIG. 4 Mode Chart for the Ferrite-Filled Waveguide at 11 GHz (other constants as for Fig. 3)

LATE NEWS RELEASES